wooden troughs kept well painted with thin coats of asphalt have given good satisfaction. They are much to be preferred to lead pipes, which continually give trouble from clogging. In concrete construction the writer has these troughs replaced by trough-like depressions made in the floors and lined with asphalt. Care should be had to make these of sufficient capacity and fall; they are covered with slate or cast-iron slabs. The vertical drains should be constructed of hard baked Akron tile or better yet, chemical pottery, and the joints made with cement or possibly with the same material as the asphalt floors. These vertical drains can either be in the elevator well or in a square space in the wall, it being closed with doors so that they too, are readily accessible. Individual traps and vents are not needed in the various laboratories, but the whole system should be effectively protected by traps in the basement. For sinks the ordinary round stoneware wash bowl may be used. This is made with an overflow, and instead of the usual brass fitting at the bottom a porcelain tube two or three inches long projects from it, carrying eyelets at the top on either side of the bowl. The tube fits down into a piece of lead pipe two feet long which empties into the trough on the back of one line of desks. This lead pipe is supported at the top by the eyelets just mentioned. These pipes can then be easily replaced by the janitor, the services of the plumber not being needed. Each laboratory should be provided with valves so that the steam, water and gas can be shut off from it without disturbing another room. The gas valve should be placed near the exit so that it can be closed nightly and diminish the danger from fire.

AUGUSTUS H. GILL
MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
March, 1909

THE PRINCIPLES OF THE CALCULUS AS APPLIED IN THE TECHNICAL COURSES OFFERED AT THE UNIVERSITY OF ILLINOIS

CONSIDERABLE discussion has been aroused in mathematical and engineering circles by the publication in SCIENCE of the papers presented at the symposium on mathematics for engineering students held in Chicago at the time of the joint meeting of the American Mathematical Society and the American Association for the Advancement of Science. The committee appointed soon after this meeting is now formulating a course in mathematics intended primarily for engineering students, and their outline will undoubtedly be accepted as a syllabus of the mathematics required by students in technical courses throughout the country. In this connection it may be suggested that some notions as to the contents of such a course might be obtained from an investigation of the various technical courses offered at some university maintaining a school of technology of recognized standing. It would be of interest to know what principles, say of the calculus, are actually used, and how often, in a single complete course or group of technical courses. Data on the relative frequency with which these principles are used might suggest the amount of emphasis to be accorded each in a course of mathematics for engineers. On the other hand, such data should also suggest to the teacher of mathematics those principles which, though not emphasized in the application, should constitute an important part of any well-rounded course in the calculus. The gaps to be thus filled become apparent on investigating what principles of the calculus are emphasized throughout the technical courses actually offered.

In this investigation I have considered the technical courses as offered in the college of engineering of the University of

Illinois and have made two assumptions which no doubt hold elsewhere.

First. Text-book work predominates throughout, and hence the texts used furnish a rational basis from which to judge both the kinds of principles used and their frequency of application.

Second. The principles, as applied in these texts, are used as often in the lectures and problems given for solution as in the texts themselves.

The list of authors of the texts studied includes the names of men connected with at least a dozen technical schools of recognized merit besides those of men at Illinois. The emphasis placed on the principles of the calculus will, necessarily, vary with the instructors in charge of the different courses; yet, as giving an average, the above assumptions seem reasonably accurate.

Any investigation of this nature will undoubtedly show that algebra and trigonometry are used much oftener than the calculus; in fact, in the comparison of numbers it would seem that the calculus plays but a minor rôle. In enumerating the principles of the calculus four complete courses were investigated. These are architectural, civil, mechanical and electrical engineering. Similar courses offered, such as municipal and sanitary, railway, civil and others, furnish results in every way analogous to those above mentioned. The architectural engineering course is included

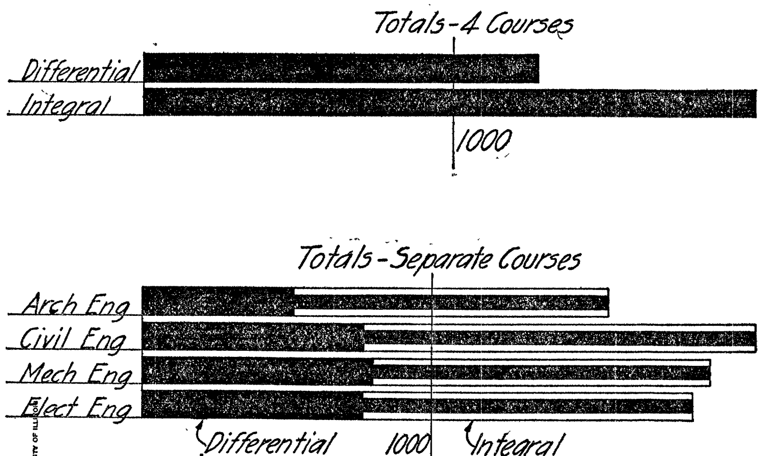
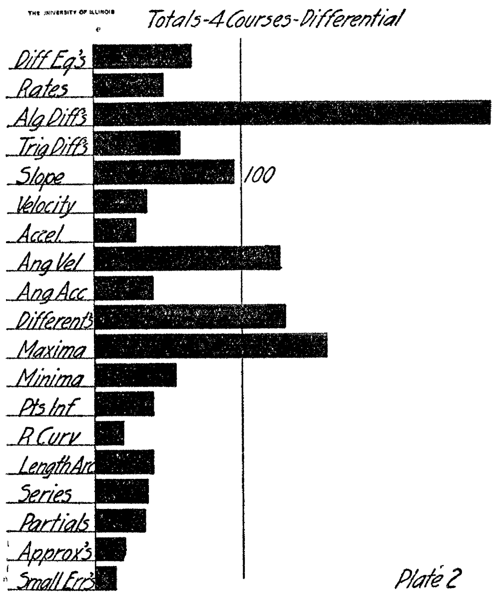


Plate 1

because in it, as offered at Illinois, are included those courses which form the backbone of all the subjects which make use of the calculus. These courses vary all the way from that in architecture, where mechanics is taught without a course in the calculus, to those in which the mathematics used furnishes the most serious difficulties met. If then a summary is made of the principles used in the four courses mentioned we have the results, as shown in Plate 1. This shows the minimum number of times a student in any of the courses listed might reasonably expect to encounter the various principles of the differential and integral calculus. Plate 1 gives us quantitative results; a qualitative analysis is given in Plates 2 and 3.

Concerning the notions of the differential calculus used it can be said that the differentiations, both algebraic and trigonomet-

ric, are almost always of the simpler sorts; in fact, as compared with algebra employed, they are very easy. Difficult differentiations occur rarely, while the trigonometric are usually limited to combinations involving sines and cosines.

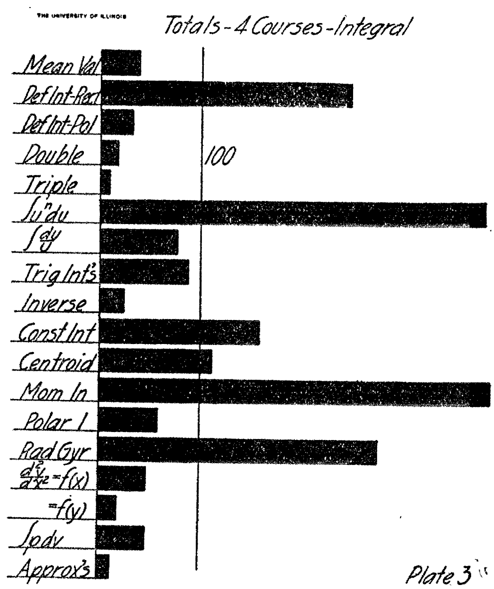


The use of differentials almost invariably brings in the notion of infinitesimals, and it is in this connection that the instructor of mathematics in preparing the future engineer can do an excellent work in giving clear notions of the differential, which need in no wise antagonize the use of the infinitesimal.

The engineering texts are certainly addicted to a rather loose use of the notion of the derivative curves corresponding to the elastic curve for concentrated loadings. A common tangent to these elastic curves at points of discontinuity of the derivative curves is frequently mentioned, and quite erroneously.

Increments and differentials are often used quite synonymously, limits are rarely mentioned, though understood to exist throughout. The duty of the mathematician is clear here, but rigor should in every case add to clearness of concept.

The subject of maxima and minima is handled without the aid of the second derivative, the nature of the problem and result being sufficient in almost every case

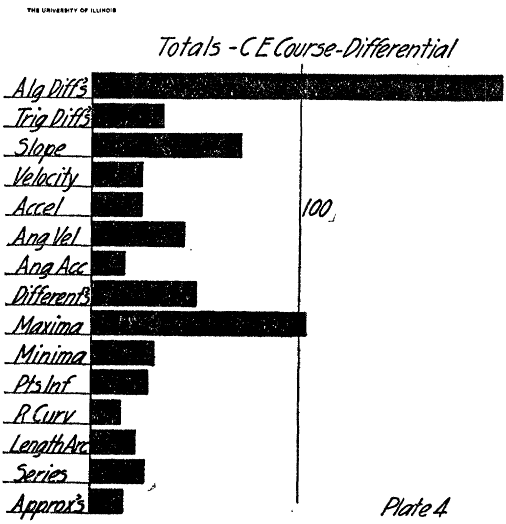


for the determination of the complete solution. Maxima and minima are often found when no first derivative is equated to zero; in fact, many cases arise where the notions exist quite sub rosa, because rigid conditions for maxima and minima can, of necessity, exist but approximately. Relative maxima and minima are emphasized and lead to a term such as maximum maximorum. It might be well for the instructor of mathematics to emphasize this feature more in his teaching. Maxima and minima are often solved from the standpoint of algebra and trigonometry.

The series found are usually simple in construction and the question of their convergence is mentioned in but a single text.

The integrals used in almost all cases arise as a consequence of the summation

process, as would naturally be expected, and the integral is most frequently that of a power of the variable or function. Double and triple integrals occur but rarely, not but what they could be used



much oftener—but they aren't. The integrals found are simple and the trigonometric are usually limited to combinations of sines and cosines.

The symbol of summation occurs frequently, while the limit of the same is rarely mentioned.

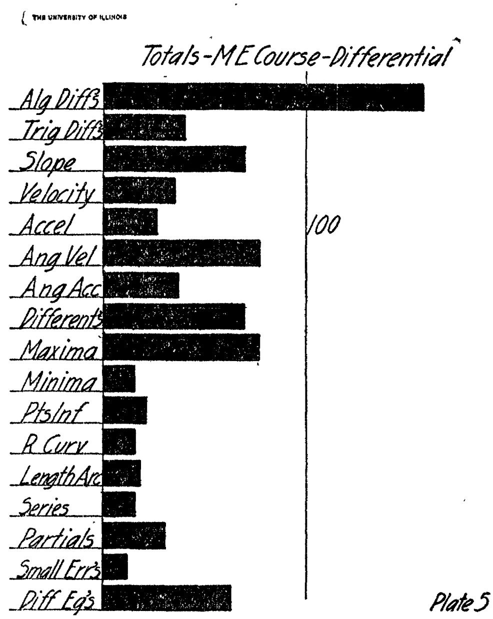
The constant of integration quite often composes the greater part of the indefinite integral, in both size and importance.

Calculus notions, such as moment of inertia, centroid and radius of gyration, occur so frequently that there are portions of certain courses which use very little else in the way of calculus. The question may be raised as to whether these should be classified as principles of the calculus or of mechanics, and the answer to this is that the various texts in the technical subjects assume that the student has learned them in the study of the calculus as a prerequisite.

The principle of mean value as used to find mean pressures, forces, etc., is restricted to the M.E. and E.E. courses.

The limits of the definite integrals used are generally quite apparent from the nature of the problem. The mathematician may well learn a lesson here in the art of making his problems both practical and concrete.

A comparison of Plates 4 and 5 will show that the matter of partial differentiation is one in which the mechanical engineer alone seems to be interested. In this connection it might also be mentioned that the principle of exact or inexact differentials, otherwise known as the integrability condition, plays quite an important and definite part in certain discussions, and that it is deserving of more attention than it receives at the hands of the authors of texts in the calculus.

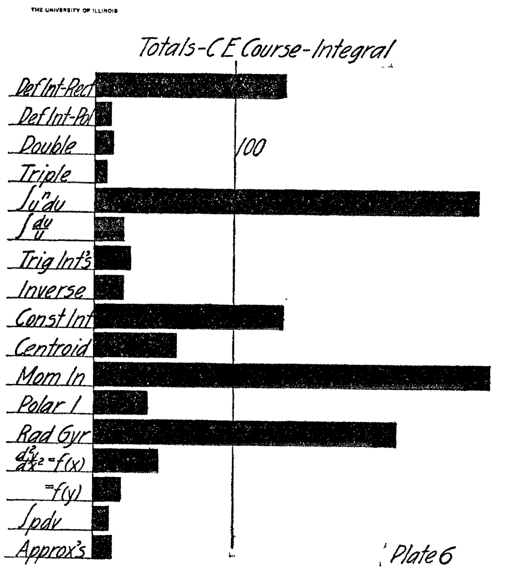


It will also be seen that the subject of angular velocities and accelerations should receive more attention than is usually given to it.

The study of differential equations, espe-

cially from the standpoint of their interpretation, is a feature of the M.E. and E.E. courses.

Plates 6 and 7 seem to indicate that the M.E. course requires a more general use

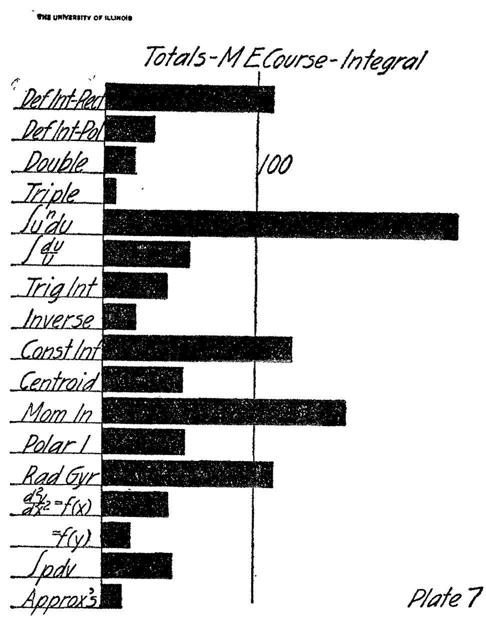


of the principles of the integral calculus than the C.E. course.

If we look for such articles as the evaluation of indeterminate forms, complicated integrals to be broken up into partial fractions before integrating, long reduction formulas, fancy substitutions, forms of remainders, order of contact, envelopes, subtangents and the like, we won't find them. Many curves are used and should be studied more for their properties than they are, one reason why they are not being the fact that the equations of so many of the curves which arise can at best be found only empirically.

It might seem that the matter of approximations is neglected in the enumeration because of the fact that so many approximations must necessarily occur in engineering practise; but in the list of those enumerated none were included which did not have a strong flavor of the calculus, which fact excluded many often listed under the head of approximate integrations, such as the use of Simpson's rule and the like.

Can we learn any general lessons from the results of the investigation? Easily. It is apparent to even the casual observer that the subject which stands out most prominently is that of the formulation of the definite integral with its limits. Here it is a question of whether or not the student can think mathematically, whether he is alive to the situation and grasps the problem before him, and whether he can express existing conditions in mathematical language. He should know the fundamental principle of the integral calculus well, and should have a check on his work wherever possible. A planimeter should find its place in the same class-room with the slide rule, and both should be used as early as possible for checking up results.



It is not sufficient to have theory only, for engineering by its very nature calls for results. The notions of the calculus are not used blindly, each has its specific ap-

plication, and the student must at all times be alive to the situation before him. It is the province of the mathematician to point out the limitations placed upon the use of the principles, not in the spirit of criticism, but of mutual help; for approximations must come into the work of the engineer, and a lot of the calculus used must be of the rough and ready sort. If the treatment can not be rigorous at all times it is the province of the mathematician to point out just how far the engineer may go and how near ideal conditions he is working—not to suggest that the whole structure is built on an insecure foundation. The engineer and mathematician can help one another, on the side of the engineer in presenting live problems in which the mathematician should be interested, and on the side of the mathematician in helping put the whole subject on a safe foundation; both working with the spirit of mutual assistance toward the doing of things worth while, not only to the engineer, but also to the mathematician.

ERNEST W. PONZER

SCIENTIFIC NOTES AND NEWS

THE Palmer Physical Laboratory of Princeton University will be formally opened on the evening of October 22, when Dr. Elihu Thomson will give the principal address. The American Physical Society will meet at Princeton on the following day, and in the evening there will be a reception at the Nassau Club.

DR. T. W. RICHARDS, professor of chemistry at Harvard University, has been given the honorary degree of doctor of philosophy by the Czech University of Prague.

THE British Institute of Marine Engineers has awarded the Denny gold medal to Mr. W. P. Durtnall, for a paper on the generation and electrical transmission of power.

IT is proposed to celebrate the fortieth year of university teaching of Professor Enrico H. Giglioni, of Florence, by presenting him with an album containing the autograph signatures of zoologists and anthropologists throughout the world. Those who wish to join in this testimonial are requested to send their autograph to Dr. Enrico Balducci, Via Romana 19, Florence.

DR. JOSEF VON HEPPERGER, professor of astronomy at Vienna, has been appointed director of the University Observatory.

THE trustees of the Lincoln State School and Colony, at Lincoln, Ill., have provided for the establishment of a department of clinical psychology in the state institution for the feeble-minded. Dr. Edmund B. Huey, who has spent the past year in clinical study in Paris, on leave from the University of Pittsburgh, has been appointed to take charge of the new department, and has begun his work at Lincoln.

PROFESSOR R. S. TARR, of the department of geology of Cornell University, has sailed for Europe, where he will spend a year on sabbatical leave.

NEWS has been received from Dr. T. G. Longstaff to the effect that he has arrived at Leh, in Ladak, after having connected the Tarim river with the Saichar glacier.

MR. SHACKLETON has left England on a continental tour, and is to tell the story of his Antarctic expedition in the principal cities of Europe. On October 9 he was to be the guest of the Royal Geographical Society at Copenhagen. He will proceed to Stockholm and Christiania, and afterwards will visit Brussels, Antwerp, Berlin, Rome, Vienna and Paris. In March he leaves England for America on an extended tour.

THE program for the meeting of the American Mathematical Society on Saturday, October 30, will include a paper by Professor Carl Runge, Kaiser Wilhelm professor at Columbia University, on "A hydrodynamic problem treated graphically."

THE faculty of fine arts of Columbia University announces a series of four lectures to be given on Monday afternoons at 4:10 o'clock in Havemeyer Hall, by Charles E. Pellew,